CRITERIA FOR THE CHOICE OF FLOOD ROUTING METHODS IN NATURAL CHANNELS WITH OVERBANK FLOWS

Roger Moussa[1]

Abstract: The classification of river waves as gravity, diffusion, or kinematic waves, corresponds to different forms of the momentum equation in the Saint-Venant system. For a given river wave, the choice of a numerical method of resolution, space and time steps to be retained, depend essentially on the form of flood hydrographs and the hydraulic properties of the river. This paper is an investigation into these areas for flood routing in natural channels with overbank flow that generally occur in wetland areas. Two sets of criteria are proposed, first to define parameter ranges representing each wave type, then, in the particular case of the diffusive and kinematic wave models, to define criteria for the choice of numerical algorithm and adequate space and time steps. The first step was an analysis of the different terms in the Saint-Venant equations as function of the balance between friction and inertia. The second part discussed questions related to the diffusive and kinematic wave models and to numerical instabilities. The technique was applied to flood routing simulation on the Loire river in France. Comparisons between results show the efficiency of the technique to analyse the Saint-Venant equations for routing with overbank flow.

Keywords : Flood routing, Overbank, Saint-Venant equations, Diffusive wave

INTRODUCTION

Many hydraulic and hydrologic problems involve the computation of the propagation of flood waves in open channels based on the solution of the well-known Saint-Venant (1871) equations. The Saint-Venant equations are coupled hyperbolic partial differential equations that cannot be solved analytically. Various numerical schemes have been proposed to resolve these equations including the

[1] Institut National de la Recherche Agronomique, Laboratoire d'étude des Interactions Sol-Agrosystème-Hydrosystème, UMR LISAH Agro.M/INRA/IRD, 2 Place Pierre Viala, 34060 Montpellier Cedex 1, France ph +33 4 99 61 24 56, fax +33 4 67 63 26 14, moussa@ensam.inra.fr

method of characteristics, a variety of sophisticated finite difference methods (Remson & Hornberger, 1971; Dooge et al., 1982, Moussa & Bocquillon, 1996b, 2001) and finite element schemes (Szymkiewicz, 1991). Comparisons of numerical solutions for the Saint-Venant equations have been published by Greco & Panattoni (1977). The complexity inherent in the calculations has led to the development of approximate methods. Several authors have examined the approximation zone of the Saint-Venant equation in the particular case of one main channel river (Ponce & Simons, 1977; Daluz Vieira, 1983; Napiorkowski, 1992; Moussa & Bocquillon, 1996a). Within this basic model, river waves may be classified as gravity, diffusion, or kinematic waves, corresponding to different forms of the momentum equation.

However, the major parts of research in this field have studied the case of a channel with one section corresponding to the main channel, and little attention was given to overbank flow during flood events. The cross section of a channel may be composed of several distinct subsections with each subsection different in roughness from the others. For example, an alluvial channel subject to seasonal floods generally consists of a main channel and two side channels.

The object of this paper is to develop a quantitative method for identifying river wave types in the case of flood events with overbank flow. The analysis presented herein endeavours to apply the theory of linear stability to the set of the Saint-Venant equations in a non-dimensionalise space as proposed by Moussa and Bocquillon (1996a, 2000), then, in the particular case of the diffusive wave equation, to guide the user in the choice of finite difference algorithm and to specify the error introduced by the retained computation algorithm.

THE SAINT-VENANT EQUATIONS FOR CHANNELS OF COMPOUND SECTION

The dynamic modelling of a one-dimensional gradually varied unsteady flow in open channels is based on the numerical solution of the Saint-Venant equations. In the case of a river with a flooded area, let W_1 and W_2 be respectively the width of the main channel and the flooded area zone respectively (Fig. 1). The two equations, describing mass and momentum, can be written as follows

$$W_2 \frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} + g.(S_f - S) = 0 \tag{2}$$

where y is the flow depth (m), V is the flow velocity (ms^{-1}), g is the acceleration due to gravity (ms^{-2}), S is the river bed slope, S_f the slope of energy line, Q the discharge (m^3s^{-1}), x is longitudinal distance (m) and t is time (s). The basic assumption to derive this system is that the flow is one-dimensional in the main channel and the flooded area, and that there are no lateral inflows or outflows.

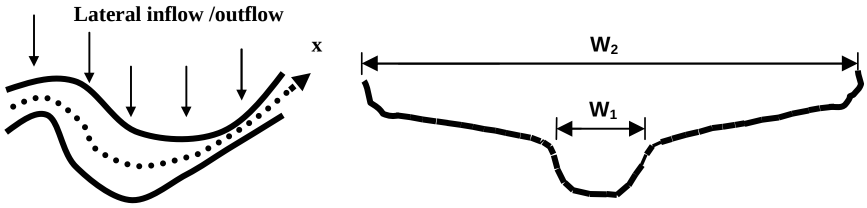

Figure 1. A channel consisting of one main section and two side sections with lateral inflow or outflow.

The side channels are usually found to be rougher than the main channel. So the mean velocity V in the main channel is greater than the mean velocities in the side channels. In such a case, the Manning formula may be applied separately to each subsection in determining the mean velocity of the subsection. Then, the discharges in the subsections can be computed. The total discharge is, therefore, equal to the sum of these discharges. As the velocity in the main channel is greater than the velocity in the flooded area, the part of discharge in the flooded area subsection is small in comparison to the discharge in the main channel. Let η be the ratio between the flooded area zone width and the main channel width

$$\eta = \frac{W_2}{W_1} \tag{3}$$

The two equations (1) and (2) of the Saint-Venant system can be written as follows (Moussa and Bocquillon, 2000)

$$\eta\frac{\partial y}{\partial t} + y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = 0 \tag{4}$$

$$\text{Term} \quad (I) \quad (II) \quad (III)$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g.(S_f - S) = 0 \tag{5}$$

$$\text{Term} \quad (IV) \quad (V) \quad (VI) \quad (VII)\,(VIII)$$

The term S_f is usually calculated using the Manning formula. As the velocity V in the main channel is greater than the velocity in the flooded area, the term of the Manning formula applied to the flooded area is small in comparison to the term in the main channel

$$S_f = n.V^2.R^{-m} \tag{6}$$

where V is the mean velocity (ms^{-1}), R the hydraulic radius (m), n the coefficient of roughness and m a constant ($m \approx 4/3$). For the main channel

$$R = \frac{W_1.y}{W_1 + 2.y} \tag{7}$$

The two equations (4) and (5) give the generalised form of the Saint-Venant system with a flooded area and where the side channels are rougher than the main channel. In this case, the Saint-Venant system depends on the parameter η that appears in the mass equation.

ANALYSIS OF RIVER WAVE TYPE

The analysis presented herein endeavours to apply the theory of linear stability to the set of equations governing the motion in open channel flow as proposed by Ponce & Simons (1977) and Napiorkowski (1992) and then to define parameter ranges representing each wave type in the Saint-Venant system for different values of η. The analysis is based on the principle that the balance between friction and inertia determines river wave behaviour.

The Saint-Venant equations are written in dimensionless form. The system equation provides parameters that quantify the magnitudes of all terms in the equation and indicate the relative importance of friction and inertia. Moussa & Bocquillon (1996a, 2000) showed that the Saint-Venant system can be expressed function of three non-dimensionalised terms :

- The Froude number $F_0^2 = \frac{V_0^2}{g y_0}$ where y_0 and V_0 are respectively the flow depth (m) and the flow velocity (ms^{-1}) of the unperturbed steady uniform flow.
- The period of the sinusoidal perturbation $T_+ = \frac{T.V_0.S_{f0}}{y_0}$ where S_{f0} is the slope of energy line of the unperturbed steady uniform flow and T the wave period (s).
- The ratio between the flooded area and the main channel widths η.

The three terms of the mass equation (4) are generally of the same order and no simplifications could be made. In opposite, and for many cases, different terms of the momentum equation (5) may be sufficiently small to be neglected, leading to further simplifications. In equation (5), the term (IV) represents the local inertia term, the term (V) represents the convective inertia term, the term (VI) represents the pressure differential term, and the terms (VII) and (VIII) accounts for the friction and bed slopes. Various wave models can be construed, depending on which of these four terms is assumed negligible when comparing with the remaining terms. Wave models and terms used to describe it are :

- Gravity wave : terms (IV) + (V) + (VI)
- Diffusive wave : terms (VI) and (VII)
- Kinematic wave : term (VII)

Fig. 2 shows the approximation zones of the Saint-Venant equations obtained by neglecting the terms of equation (5) representing less than 10 % of the sum of absolute values of all terms of this equation. Two cases are analysed, the first corresponds to the case without overbank flow (η = 1; Fig. 2a) and the second to the case with overbank flow (e.g. η = 20; Fig. 2b). The results obtained for η = 1 are of the same order and confirm other results obtained by several researchers, notably Daluz Vieira (1983), that developed criteria for deciding the conditions under which approximate models provide an acceptable representation of the momentum equation in the Saint-Venant system. When η increases, the domain of application of the gravity wave increase while the domain of application of the diffusive wave and the kinematic wave models is restricted and substituted by the full Saint-Venant system. The comparison of the diffusive wave domain of application for the two cases of η in Fig. 2, shows that the domain moves to the right (higher values of T_+) when η increases and substitute for the kinematic wave zone.

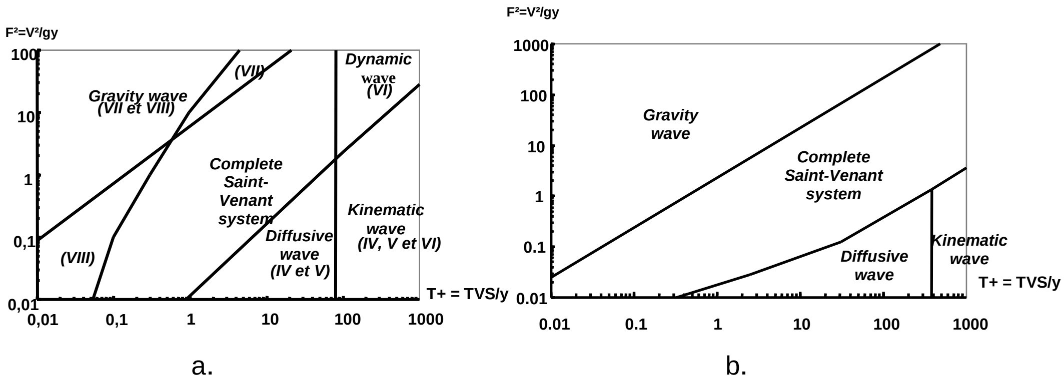

Figure 2. River wave approximation zones obtained from the analysis of the momentum equation of the Saint-Venant system for two values of η =1 (a) and 20 (b).

Other considerations such as the computational power available or the need for real-time forecasting may also be important in the choice of technique. Practical experience suggests that the simpler diffusive wave, kinematic wave or linear methods will be adequate for many purposes. They will not, however, be suitable in flood routing modeling when η increases or varies in space (on the river channel) or in time (during a flood event). In this case, the diffusive wave model should substitute for the kinematic wave model and the full Saint-Venant system should substitute for the diffusive wave model.

In choosing a routing method the accuracy and availability of channel cross-section and roughness coefficients may have a greater effect on the predictive accuracy of a routing algorithm than the choice of the descriptive equation. In addition, both cross-sectional and reach scale roughness may be expected to vary with discharge, especially at the transition overbank flow. Estimating roughness coefficients is a particularly difficult problem for natural channels. In fact, all routing methods will need to be calibrated to a particular site by comparing observed and predicted levels or discharges, where they are available. In general, the more complex the model, the more physical characteristics and model parameters that must be estimated or measured in the field. In this respect, the simpler routing methods with fewer parameters may have some advantages.

THE DIFFUSIVE WAVE MODEL

In most practical applications, the acceleration terms in the momentum balance of the Saint-Venant equations can be neglected since they are small in comparison to the channel bed slope. We obtain the diffusive wave equation

$$\frac{\partial Q}{\partial t}+C\left(\frac{\partial Q}{\partial x}-q\right)-D\left(\frac{\partial^2 Q}{\partial x^2}-\frac{\partial q}{\partial x}\right)=0 \tag{8}$$

where x (m) is downstream distance, t (s) time, C (ms^{-1}) and D (m^2s^{-1}) are non-linear functions of the discharge Q(x,t) (m^3s^{-1}) and are generally known as celerity and diffusivity, respectively. The kinematic wave model corresponds to the particular case when D=0. The term q(x,t) (m^2s^{-1}) represents the lateral inflow distribution (lateral inflow if q>0, and lateral outflow if q<0). Let $Q_a(t)$ (m^3s^{-1}) be the total lateral inflow/outflow hydrograph.

$$Q_a(t)=\int_0^L q(x,t).dx \tag{9}$$

The diffusive wave approximation, like the full Saint-Venant system, requires specification of both upstream and downstream boundary conditions, as well as patterns of lateral inflows. It can be used to model both the attenuation of the flood wave peak downstream and backwater effects resulting from obstructions to the flow and the confluence of different branches of the network (Dooge et al. 1982). In practice, attenuation and dispersion of the flood wave may be masked by significant lateral inflow.

For a river reach without lateral inflow or outflow, the diffusive wave has an analytical solution : The Hayami model (1951)

$$Q(x,t)=Q(0,0)+\frac{x}{2(\pi.D)^{\frac{1}{2}}}.\exp^{\frac{C.x}{2.D}}\int_0^t (Q(0,t-\tau)-Q(0,0)).\frac{\exp^{-\frac{C.x}{4.D}\left(\frac{x}{C.\tau}+\frac{C.\tau}{x}\right)}}{\tau^{\frac{3}{2}}}d\tau \tag{10}$$

for $0\le x\le L$

Let I(t) and O(t) be respectively the upstream inflow minus the baseflow and the downstream outflow minus the baseflow, we have

$$I(t)=Q(0,t)-Q(0,0) \qquad O(t)=Q(L,t)-Q(L,0) \tag{11}$$

and let K(t) be the Hayami kernel function defined as

$$K(t) = \frac{L}{2(\pi D)^{\frac{1}{2}}} \frac{\exp^{\frac{CL}{4D}\left(2-\frac{L}{Ct}-\frac{Ct}{L}\right)}}{t^{\frac{3}{2}}} = \left(\frac{\theta.z}{\pi}\right)^{\frac{1}{2}} \frac{\exp^{z\left(2-\frac{\theta}{t}-\frac{t}{\theta}\right)}}{t^{\frac{3}{2}}} = K_{\theta,z}(t) \qquad (12)$$

with $$\theta = \frac{L}{C} \qquad z = \frac{C.L}{4.D} \qquad (13)$$

Substituting (11), (12) and (13) into (10) gives the convolution relation

$$O(t) = \int_0^t I(t-\tau).K(\tau).d\tau = I(t) * K(t) \qquad (14)$$

In the particular case where lateral flow is uniformly distributed along the river reach, Moussa (1996) showed that the diffusive wave model has an analytical solution

$$O(t) = \Phi(t) + \left(I(t) - \Phi(t)\right) * K(t) \qquad (15)$$

where $$\Phi(t) = \frac{C}{L}\int_0^t \left(Q_a(\lambda) - Q_a(0)\right) d\lambda \qquad (16)$$

The analytical solution is unconditionally stable, requires only an upstream boundary condition, which makes it very convenient to implement for networks of channel reaches and has the benefit of ending as a very simple computer program (Moussa, 1997). However when the parameters C and D are function of Q, numerical methods are required to resolve the diffusive wave equation (Moussa & Bocquillon, 1996b, 2001). When using numerical solution, one encounters the questions of construction of finite-difference systems, methods for solving them, their stability and their accuracy. The choice of an algorithm, time and space steps to be retained, depend on many factor : the form of upstream flood hydrograph, the hydraulic properties of river reaches and the recording time step. Moussa and Bocquillon (1996a) proposed a methodology to optimise the choice of time and space steps of finite difference methods in order to avoid instabilities and to reduce the time of calculation.

APPLICATION CASE

The River Loire in France between Grangent and Feurs (L = 43 km) was used as application case. The lateral subcatchment (area 865 km²) is equipped with two stream recorders that give upstream inflow at Grangent (catchment area 4113 km²) and downstream outflow at Feurs (catchment area 4978 km²), the mean annual discharge at Feurs being 41.4 m^3s^{-1}. The river elevation ranges from 373 m above sea level at Grangent to 325 m above sea level at Feurs. The mean width of the river is W_1=130m, the mean width of the flooded area W_2=200m to 1000 m and the mean slope of the river bed slope is S=0.1 %. By comparing the relations flow depth / discharge at the two control sections (approximately 3.4 m and 5.8 m flow depths for discharges of 500 m^3s^{-1} and 1200 m^3s^{-1}respectively), a mean velocity was estimated; the calculated values range between 1.1 ms^{-1} and 1.4 ms^{-1} during flood events. Six inflow/outflow hydrographs measured by EPALA (Etablissement Public d'Aménagements de la Loire et ses Affluents) where lateral inflow could be neglected were considered between 1976 and 1984. The time step is two hours. For all flood events, the maximum peak of outflow is inferior to the maximum peak of inflow reflecting the presence of diffusivity. The aim is to propose a methodology to guide the user in the choice of a flood routing model that simulates outflow at Feurs using as inputs measured inflow at Grangent and then to choose numerical algorithm and space and time steps that minimise errors criteria defined by the user.

The analysis of inflow hydrographs shapes was done by adjusting a sinusoidal function. Then the problem was analysed in the Saint-Venant approximation zone diagram (Fig. 2) by calculating the Froude number and the wave characteristics of each flood event. For the six flood events, the Froude number ranges in the interval $0.03 \le F_0^2 \le 0.12$ and the nondimensionalized period in the interval $10 \le T_+ \le 45$. The parameter η ranges between 1 and 8. This corresponds to the diffusive wave zone.

The methodology proposed to analyse the diffusive wave routing problem is divided into two steps. The first step was to estimate, for each set of input-output hydrographs the two parameters C and D of the diffusive wave equation (8) by analysing inflow-outflow hydrographs and geometric characteristics of the river under the hypothesis of constant values of C and D. The second step was to optimise the choice of the numerical algorithm and the choice of space and time steps that minimise numerical errors. For this the analytical solution (Eq. 15) was compared to the solution obtained with numerical methods suggested by Moussa & Bocquillon (1996b). Two problems were studied: For a given algorithm and for a given space and time steps, what is the numerical error induced? What algorithm and what space and time steps should be used to obtain a numerical error less then a threshold defined by the user?

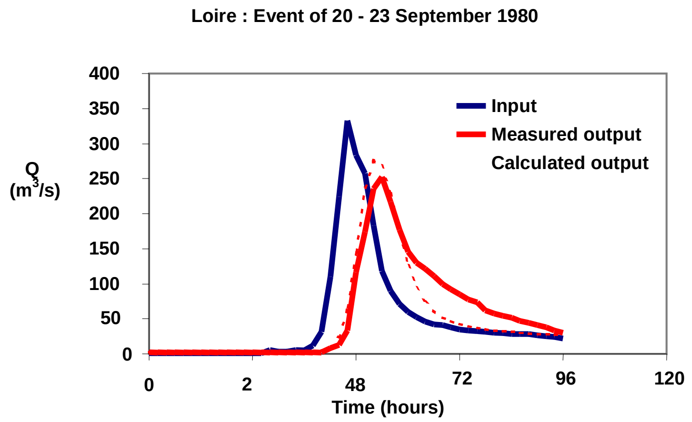

Figure 3. Comparison between measured and calculated hydrographs for a flood event on the Loire river between Grangent and Feurs.

For the application case, the identified parameters C and D for the six flood events ranges for the celerity in the interval $1.3\,\mathrm{m/s} \leq C \leq 2.8\,\mathrm{m/s}$ and for the diffusivity in the interval $1500\,\mathrm{m^2/s} \leq D \leq 13200\,\mathrm{m^2/s}$. The algorithm that reduces the calculation time and numerical instabilities was the Crank-Nicholson scheme with Δx = 7166 m and Δt = 1440 s. Fig. 3 shows a comparison between observed discharge and simulated discharge for one of the six flood events on the Loire river using the adjusted parameters of the diffusive wave equation with overbankflow.

CONCLUSION

The Saint-Venant system, formed by combining the continuity and momentum equations, is controlled by the balance between friction and inertia. The case of compound channel section is studied for flood routing problems in natural channels with overbank flow in the flooded area. The propagation characteristics of shallow water waves in open channel flow are calculated on the basis of linear stability theory. Two dimensionless parameters, the Froude number and the period of the input hydrograph enable to define a spectrum of river waves, with continuous transitions between wave types. Gravity, diffusion, and kinematic waves correspond to specific scaling parameter ranges of this spectrum. The parameter range corresponding to each wave type was studied as a function of the flooded area width. Results show that, when the width of the flooded area increases, the domain of application of the diffusive wave and the kinematic wave models is restricted.

Then, in the particular case of the diffusive wave model, a methodology is proposed to guide the user in the choice of numerical algorithm and the adequate space and time steps. This methodology, based upon an analysis of inflow/outflow hydrographs, enables the user to estimate error induced by the numerical algorithm and to optimise space and time steps that minimise criteria errors and computer time. This technique was applied for the Crank-Nicholson algorithm on the Loire river as demonstration case. However, the same methodology could be used with other algorithms needing other set of parameters.

REFERENCES

Daluz Vieira J.H., 1983 - *Conditions governing the use of approximations for the Saint-Venant equations for shallow water flow*. Journal of Hydrology, 60, 43-58.

Dooge J.C.I., Strupczewski W.B., Napiorkowski J.J., 1982 - *Hydrodynamic derivation of storage parameters of the Muskingum model.* Journal of Hydrology, 54, 371-387.

Greco F., Panattoni L., 1977 - *Numerical solutions methods of the Saint-Venant equations*. In : Ciriani T.A., Maione, U., Wallis J.R. (eds) - *Mathematical models for surface water hydrology*.,Wiley, Chichester, 181-194.

Hayami S., 1951 - *On the propagation of flood waves*. Disaster Prev. Res. Inst. Bull., 1: 1-16.

Moussa R., 1996 - *Analytical Hayami solution for the diffusive wave flood routing problem with lateral inflow.* Hydrological Processes, 10(9), 1209-1227.

Moussa R., 1997 - *Geomorphological transfer function calculated from digital elevation models for distributed hydrological modelling.* Hydrological Processes, 11(5), 429-449.

Moussa R., Bocquillon C., 1996a - *Criteria for the choice of flood-routing methods in natural channels.* Journal of Hydrology, 186(1-4), 1-30.

Moussa R., Bocquillon C., 1996b - *Algorithms for solving the diffusive wave flood routing equation.* Hydrological Processes, 10(1), 105-124.

Moussa R., Bocquillon C., 2000 - *Approximation zones of the Saint-Venant equations for flood routing with overbank flow.* Hydrology and Earth System Sciences, 4(2), 251-261.

Moussa R., Bocquillon C., 2001 - *Fractional-step method solution of diffusive wave equation.* Journal of Hydrologic Engineering, 6(1), 11-19.

Napiorkowski J.J., 1992 - *Linear theory of open channel flow.* In : O'Kane J.P. (ed) - *Advances in theoretical hydrology*, Elsevier, Amsterdam, 3-15.

Ponce V.M., Simons D.B., 1977 - *Shallow wave propagation in open channel flow.* ASCE Journal of Hydraulic Division, 103(HY12), 1461-1476.

Remson R.D., Hornberger G.M., Molz F.J., 1971 - *Numerical methods in subsurface hydrology*. Willey Interscience, New York, 399 pp.

Saint-Venant B., 1871 - *Théorie du mouvement non permanent des eaux, avec application aux crues des rivières et à l'introduction des marées dans leurs lits.* Compte-rendus de l'Académie des Sciences, 73, 147-154 and 237-240 (in French).

Szymkiewicz R., 1991 - *Finite-element method for the solution of the Saint Venant equations in an open channel network.* Journal of Hydrology, 122, 275-287.